Let P be the point $(8,-5)$, and Q be the point $(-1,-2)$.
Let $\vec{m}$ be the vector with terminal point P and initial point Q.

SCORE: ____ / 24 PTS

[a] Find a unit vector perpendicular to $\vec{m}$. Write your final answer in component form.

$\vec{m} = \langle 9,-3 \rangle$

$\langle 9,-3 \rangle \cdot \langle a,b \rangle = 0$

$9a - 3b = 0$

$a=1, b=3$

$\frac{1}{\|\langle 1,3 \rangle\|} \langle 1,3 \rangle = \frac{1}{\sqrt{10}} \langle 1,3 \rangle$

$= \left\langle \frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10} \right\rangle$

[b] A force represented by the vector $\langle 2,-4 \rangle$ moves an object from Q to P. Find the work done.

$\langle 2,-4 \rangle \cdot \langle 9,-3 \rangle = 18 + 12 = 30$

If $\vec{r} = 3\vec{j} - 2\vec{k}$ and $\vec{s} = 5\vec{i} - 4\vec{j} + c\vec{k}$ are orthogonal, find the value of c.

SCORE: ____ / 10 PTS

$\vec{r} \cdot \vec{s} = 0$

$0(5) + 3(-4) - 2(c) = 0$

$-12 - 2c = 0$

$c = -6$

MULTIPLE CHOICE: **Circle the correct answer.**

SCORE: ____ / 9 PTS

If $\|\vec{x}\| = 7$, $\|\vec{y}\| = 6$ and the angle between $\vec{x}$ and $\vec{y}$ is $\frac{5\pi}{6}$, then $\vec{x} \cdot \vec{y} =$ $\|\vec{x}\| \|\vec{y}\| \cos \frac{5\pi}{6}$

[a] -21 [b] $21\sqrt{2}$ [c] $-21\sqrt{3}$ [d] 42 [e] none of the above

Let D be the point $(3,-1,-2)$, E be the point $(1,3,-1)$ and F be the point $(2,-2,1)$. **SCORE: ____ / 42 PTS**

[a] Find the area of the triangle DEF.

$\overrightarrow{DE} = \langle -2,4,1 \rangle$

$\overrightarrow{DF} = \langle -1,-1,3 \rangle$

$$\overrightarrow{DE} \times \overrightarrow{DF} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 4 & 1 \\ -1 & -1 & 3 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ -2 & 4 \\ -1 & -1 \end{matrix} = 12\vec{i} - \vec{j} + 2\vec{k} + \vec{i} + 6\vec{j} + 4\vec{k}$$

$$= \langle 13,5,6 \rangle$$

$$\frac{1}{2}\|\langle 13,5,6 \rangle\| = \frac{1}{2}\sqrt{230}$$

[b] Find the general form of the equation of the plane containing D, E and F.

$$13(x-3) + 5(y+1) + 6(z+2) = 0$$

$$13x + 5y + 6z = 22$$

[c] Find the symmetric equation of the line through E and parallel to the line with parametric equations $x = 13t - 15$, $y = 21t + 11$, $z = 17 - 19t$.

$$\frac{x-1}{13} = \frac{y-3}{21} = \frac{z+1}{-19}$$

DIRECTION VECTOR $\langle 13, 21, -19 \rangle$

[d] Find parametric equations of the line through F and perpendicular to the plane $7x - 9z = 4$.

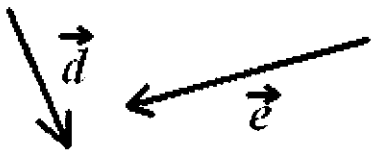
$$x = 2 + 7t$$

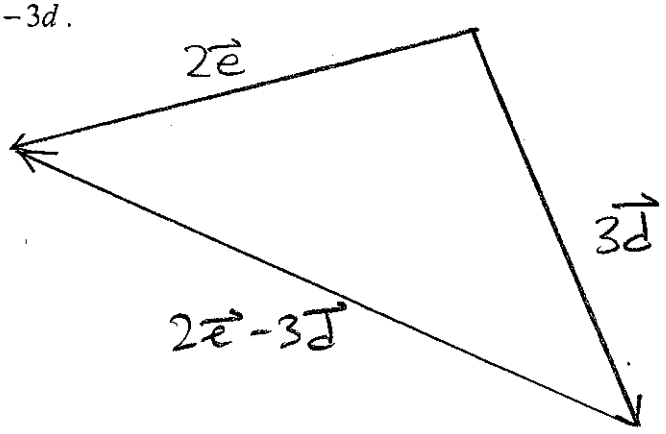
$$y = -2$$

$$z = 1 - 9t$$

NORMAL VECTOR $\langle 7, 0, -9 \rangle$

Vectors $\vec{d}$ and $\vec{e}$ are shown below. Sketch $2\vec{e} - 3\vec{d}$. **SCORE: ____ / 10 PTS**





Let $\vec{p} = -3\vec{j} + \vec{k}$ and $\vec{q} = -\vec{i} - 2\vec{j} + 2\vec{k}$.

SCORE: ____ / 36 PTS

[a] Find a vector of magnitude 5 in the opposite direction as $\vec{q}$.

$$-\frac{5}{\|\vec{q}\|}\vec{q} = \frac{-5}{3}\langle -1, -2, 2\rangle = \left\langle \frac{5}{3}, \frac{10}{3}, \frac{-10}{3} \right\rangle$$

[b] Write $\vec{p}$ as the sum of two orthogonal vectors, one of which is $proj_{\vec{q}}\vec{p}$.

$$\text{PROJ}_{\vec{q}}\vec{p} = \frac{\vec{p}\cdot\vec{q}}{\vec{q}\cdot\vec{q}}\vec{q}$$

$$= \frac{0+6+2}{9}\langle -1, -2, 2\rangle$$

$$= \frac{8}{9}\langle -1, -2, 2\rangle$$

$$= \left\langle \frac{-8}{9}, \frac{-16}{9}, \frac{16}{9} \right\rangle$$

$$\langle 0, -3, 1\rangle - \left\langle -\frac{8}{9}, -\frac{16}{9}, \frac{16}{9} \right\rangle$$

$$= \left\langle \frac{8}{9}, \frac{-11}{9}, \frac{-7}{9} \right\rangle$$

$$\vec{p} = \left\langle -\frac{8}{9}, -\frac{16}{9}, \frac{16}{9} \right\rangle + \left\langle \frac{8}{9}, \frac{-11}{9}, \frac{-7}{9} \right\rangle$$

[c] Find a vector perpendicular to both $\vec{p}$ and $\vec{q}$.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -3 & 1 \\ -1 & -2 & 2 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 0 & -3 \\ -1 & -2 \end{matrix} = -6\vec{i} - \vec{j} + 2\vec{i} - 3\vec{k} = \langle -4, -1, -3\rangle$$

If $\vec{g} = a\vec{i} - 9\vec{j} + 4\vec{k}$ and $\vec{h} = 5\vec{i} - 6\vec{j} + b\vec{k}$ are parallel, find the values of a and b.

SCORE: ____ / 10 PTS

$$\vec{g} = c\vec{h}$$

$$\langle a, -9, 4\rangle = c\langle 5, -6, b\rangle$$

$$a = 5c \longrightarrow a = 5\left(\tfrac{3}{2}\right) = \tfrac{15}{2}$$

$$-9 = -6c \longrightarrow c = \tfrac{3}{2}$$

$$4 = bc \longrightarrow 4 = b\left(\tfrac{3}{2}\right) \longrightarrow b = \tfrac{8}{3}$$

MULTIPLE CHOICE: Circle the correct answer.

SCORE: ____ / 9 PTS

If $\vec{b}\cdot\vec{c} = -76$, then the angle between $\vec{b}$ and $\vec{c}$ **could** be

ANGLE MUST BE OBTUSE

[a] 0° [b] 14° [c] 90° [d] (166°) [e] 284°